2D Gabor functions and filters for image processing and computer vision

Nicolai Petkov
Intelligent Systems group
Institute for Mathematics and Computing Science
Most of the images in this presentation were generated with the on-line simulation programs available at:

http://matlabserver.cs.rug.nl
Neurophysiologic background
Primary visual cortex (striate cortex or V1)

Brodmann area 17

Wikipedia.org
Simple and complex cells: respond to bars of given orientation


Simple cells
and
Gabor filters
(or a Platonic view of reality)
Hubel and Wiesel named one type of cell "simple" because they shared the following properties:

- Their receptive fields have distinct excitatory and inhibitory regions.
- These regions follow the summation property.
- These regions have mutual antagonism - excitatory and inhibitory regions balance themselves out in diffuse lighting.
- It is possible to predict responses to stimuli given the map of excitatory and inhibitory regions.

In engineering terms:
a simple cell can be characterized by an impulse response.
Receptive field profiles of simple cells

How are they determined?
• recording responses to bars
• recording responses to gratings
• reverse correlation (spike-triggered average)

Why do simple cells respond to bars and gratings of given orientation?
1D:  

2D:  

2D Gabor functions
$$g_{\lambda, \Theta, \varphi, \sigma, \gamma}(x, y) = \exp \left( -\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2} \right) \cos \left( 2\pi \frac{x'}{\lambda} + \varphi \right)$$ (1)

$$x' = x\cos\Theta + y\sin\Theta$$
$$y' = -x\sin\Theta + y\cos\Theta$$
Parameterization according to:


Preferred spatial frequency (1/\(\lambda\)) and size (\(\sigma\)) are not completely independent:

\[ \sigma = a\lambda \]

with \(a\) between 0.3 and 0.6 for most cells. In the following, we use mostly \(\sigma = 0.56\lambda\).
Preferred spatial frequency and size

Space domain

Frequency domain

Wavelength = 2/512

Frequency = 512/2
Preferred spatial frequency and size

Space domain

Wavelength = 4/512

Frequency domain

Frequency = 512/4
Preferred spatial frequency and size

Space domain

Wavelength = 8/512

Frequency domain

Frequency = 512/8
Preferred spatial frequency and size

Space domain

Wavelength = 16/512

Frequency domain

Frequency = 512/16
Preferred spatial frequency and size

Space domain

Frequency domain

Wavelength = 32/512

Frequency = 512/32
Wavelength $= \frac{64}{512}$

Frequency $= \frac{512}{64}$
Orientation (θ)

\[ g_{\lambda, \theta, \varphi, \sigma, \tau}(x, y) = \exp \left( -\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2} \right) \cos \left( 2\pi \frac{x'}{\lambda} + \varphi \right) \] (1)

\[
x' = x \cos \theta + y \sin \theta
\]

\[
y' = -x \sin \theta + y \cos \theta
\]
Orientation = 0
Orientation

Space domain

Frequency domain

Orientation = 45
Orientation

Space domain

Frequency domain

Orientation = 90
Symmetry (phase offset $\varphi$)

$g_{\lambda, \Theta, \varphi, \sigma, \tau}(x, y) = \exp \left( -\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2} \right) \cos \left( 2\pi \frac{x'}{\lambda} + \varphi \right) \quad (1)$

\[
\begin{align*}
  x' &= x\cos\Theta + y\sin\Theta \\
  y' &= -x\sin\Theta + y\cos\Theta
\end{align*}
\]
Symmetry (phase offset)

Space domain

Phase offset = 0
(symmetric function)

Space domain

Phase offset = -90
(anti-symmetric function)
Spatial aspect ratio ($\gamma$)

$$g_{\lambda, \theta, \phi, \sigma, \gamma}(x, y) = \exp \left( -\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2} \right) \cos \left( 2\pi \frac{x'}{\lambda} + \varphi \right) \quad (1)$$

\[
x' = x \cos \Theta + y \sin \Theta
\]
\[
y' = -x \sin \Theta + y \cos \Theta
\]
Spatial aspect ratio

Aspect ratio = 0.5
Spatial aspect ratio

Space domain

Frequency domain

Aspect ratio = 1
Spatial aspect ratio

Space domain

Frequency domain

Aspect ratio = 2
(does not occur)
Half-response spatial frequency bandwidth $b$ (in octaves)

$$
\begin{align*}
\sigma & = \frac{1}{\pi} \sqrt{\frac{\ln 2}{2}} \cdot \frac{2^b + 1}{2^b - 1} \\
\frac{\sigma}{\lambda} & = \frac{\pi}{\lambda} + 1 \sqrt{\frac{\ln 2}{2}}
\end{align*}
$$

$$
\begin{align*}
g_{\lambda, \theta, \varphi, \sigma, \tau}(x, y) &= \exp \left( -\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2} \right) \cos \left( 2\pi \frac{x'}{\lambda} + \varphi \right) \\
x' &= x\cos\Theta + y\sin\Theta \\
y' &= -x\sin\Theta + y\cos\Theta
\end{align*}
$$
Space domain

Frequency domain

Bandwidth = 1 ($\sigma = 0.56\lambda$)

Wavelength = $8/512$
Bandwidth

Space domain

Frequency domain

Bandwidth = 0.5
Wavelength = 8/512
Space domain

Frequency domain

Bandwidth = 2

Wavelength = 8/512
Bandwidth

Space domain

Frequency domain

Bandwidth = 1 (σ = 0.56λ)

Wavelength = 32/512
Bandwidth

Space domain

Frequency domain

Bandwidth = 0.5
Wavelength = 32/512
Space domain

Frequency domain

Bandwidth = 2

Wavelength = 32/512
Semi-linear 2D Gabor filter

\[ R = |g * I|^+ \]

i.e., the response \( R \) is obtained by convolution \((*)\) of the input \( I \) with a Gabor function \( g \), followed by half-wave rectification \((|.|^+)\)
Semi-linear Gabor filter

What is it useful for?

- Receptive field $g(-x,-y)$
- Output of convolution followed by half-wave rectification
- $bw2 = 2$
- $	ext{Ori} = 0$  $	ext{Phi} = 90$  edges
- $	ext{Ori} = 180$  $	ext{Phi} = 90$  edges
- $	ext{Ori} = 0$  $	ext{Phi} = 0$  lines
- $	ext{Ori} = 0$  $	ext{Phi} = 180$  lines
Bank of semi-linear Gabor filters
Which (an how many) orientations to use?

For filters with s.a.r=0.5 and bw=2, good coverage of angles with 6 orientations
Bank of semi-linear Gabor filters

Which (and how many) orientations to use?

For filters with $\text{sar}=0.5$ and $\text{bw}=2$, good coverage of angles with 12 orientations.
Result of superposition of the outputs of 12 semi-linear anti-symmetric (phi=90) Gabor filters with wavelength = 4, bandwidth = 2, spatial aspect ratio = 0.5 (after thinning and thresholding lt = 0.1, ht = 0.15).
Bank of semi-linear Gabor filters
Which (and how many) frequencies to use?

Receptive field
\( g(-x,-y) \)

Frequency domain

Wavelength = 2 8 32 128 (s.a.r.=0.5)

For filters with \( bw=2 \), good coverage of frequencies with wavelength quadropping.
Bank of semi-linear Gabor filters

Receptive field $g(-x,-y)$

Frequency domain

Wavelength = 2  4  8  16  32

For filters with $bw=1$, good coverage of frequencies with wavelength doubling
Complex cells
and
Gabor energy filters
Simple and complex cells: respond to bars of given orientation


Hubel and Wiesel named another type of cell “complex" because they contrasted simple cells in the following properties:

• Their receptive fields do not have distinct excitatory and inhibitory regions.

• Their response cannot be predicted by weighted summation.

• Response is not modulated by the exact position of the optimal stimulus (bar or grating).

In engineering terms: a complex cell cannot be characterized by an impulse response.
Gabor energy model of a complex cells

\[ E_{\lambda, \sigma, \theta}(x, y) = \sqrt{R^2_{\lambda, \sigma, \theta, 0}(x, y) + R^2_{\lambda, \sigma, \theta, -\frac{\pi}{2}}(x, y)} \]

Phase offset = 0
(symmetric function)

Phase offset = -90
(anti-symmetric function)
Gabor energy filter

Input

Receptive field $g(-x,-y)$

Output of convolution followed by half-wave rectification

Gabor energy output

Ori = 0
Phi = 90

Ori = 180
Phi = 90

Ori = 0
Phi = 0

Ori = 0
Phi = 180
Result of superposition of the outputs of 4 Gabor energy filters (in [0,180)) with wavelength = 8, bandwidth = 1, spatial aspect ratio = 0.5
Result of superposition of the outputs of 6 Gabor energy filters (in $[0,180)$) with wavelength = 4, bandwidth = 2, spatial aspect ratio = 0.5 (after thinning and thresholding $lt = 0.1$, $ht = 0.15$).
More efficient way to detect intensity changes by gradient computation

Space domain

Frequency domain

\[ \frac{dG}{dx} \] \[ \frac{dG}{dy} \]
More efficient way to detect intensity changes

Gradient magnitude

Canny
Various ways to detect edges

Gabor filter

Gabor energy

Canny

http://matlabserver.cs.rug.nl
Gabor filters for texture analysis

Filters in frequency domain
Gabor filters for texture analysis

See e.g.

S.E. Grigorescu, N. Petkov and P. Kruizinga:
Comparison of texture features based on Gabor filters,

and references therein

http://matlabserver.cs.rug.nl
Problems with texture edges

- Gabor filter
- Gabor energy
- Canny

http://matlabserver.cs.rug.nl
Contour enhancement by suppression of texture

Canny with surround suppression

[Petkov and Westenberg, Biol.Cyb. 2003]
[Grigorescu et al., IEEE-TIP 2003, IVC 2004]
See

N. Petkov and E. Subramanian:
Motion detection, noise reduction, texture suppression and contour enhancement by spatiotemporal Gabor filters with surround inhibition,
*Biological Cybernetics*, **97** (5-6), 2007, 423-439.

and references therein

http://www.cs.rug.nl/~petkov/publications/journals
Complex cells and CORF filters

(or a non-Gaborian, less Platonic view of reality)
References to origins – modeling

2D Receptive Field

2D Gabor Function

Difference
V1 complex cells modeled by CORF: Combination Of LGN Receptive Fields

[Azzopardi and Petkov, 2011]
V1 complex cells modeled by CORF: Combination Of LGN Receptive Fields

Image

Ground truth

Gabor Energy

CORF

Precision

Recall

GE; $\sigma = 5$

CORF; $\sigma = 5$
V1 complex cells modeled by
CORF: Combination Of LGN Receptive Fields
V1 complex cells modeled by CORF: Combination Of LGN Receptive Fields
V1 complex cells modeled by
CORF: Combination Of LGN Receptive Fields
V1 complex cells modeled by
CORF: Combination Of LGN Receptive Fields

CORF is more effective than GE

- Better contour integration
- More robust to noise
- Better edge localization

(a) SNR = ∞  (b) SNR = 5  (c) SNR = 2.5
(d) CORF  
\[ F\text{-Measure} = 1 \]  
(e) CORF  
\[ F\text{-Measure} = 0.72 \]  
(f) CORF  
\[ F\text{-Measure} = 0.51 \]

(g) GE  
\[ F\text{-Measure} = 0.48 \]  
(h) GE  
\[ F\text{-Measure} = 0.43 \]  
(i) GE  
\[ F\text{-Measure} = 0.17 \]
Computational models (of V1/2) are approximations

Plato: use Gabor energy (for aesthetic reasons)

Popper: use CORF (for practical empiricism)